Flood control of river systems with Model Predictive Control

The river Demer as case study

Maarten Breckpot

Jury:
Y. Willems, chair
B. De Moor, promotor
P. Willems
M. Diehl
B. De Schutter
(TU Delft)
B. Pluymers
(IPCOS NV)

KU LEUVEN

Why is this research necessary?

- Number of heavy floodings ↑↑

	1970-1979	1980-1989	1990-1999	2000-2009
worldwide	263	526	780	1729
Europe	23	38	94	239
Belgium	1	2	4	6

- The Rhine: 400 – 500 million euro (1993)
- > 100 big floods: 25 billion euro (1998-2004),
  700 people ✞, half million homeless
- Example in Belgium: **the Demer**

KU LEUVEN

The Demer: a history of normalization and floodings

Measures taken in the past:

- Normalization
- Dikes

+ increasing urbanization in flood sensitive areas

New vision on flood control/management

- Preservation/restoration of natural flood areas
- Reservoirs
- Computer controlled management:
  advanced three-position controller

Not effective

Not effective


2002
© De Digitale Demer
BERINGEN
Heusden
Deurne
Langdorp
DIEPENBEEK
Kerkom
Roosbeek
TIENEN
Halle-Booienhoven

The Demer: a history of normalization and floodings

Objective:

Can Model Predictive Control be used for
set-point control and flood control of river systems?

⇩

Approach:

1. General modelling framework
2. Find accurate approximate model
3. Design controller

advanced three-position controller

Not effective

More intelligent flood regulation required!

Model Predictive Control?

KU LEUVEN

What is Model Predictive Control?

$$\min_{\mathbf{u},\mathbf{x}} \sum_{j=1}^{N_\mathsf{P}} \|\mathbf{x}(j) - \mathbf{r}_x(j)\|_\mathbf{Q}^2 + \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j) - \mathbf{u}(j-1)\|_\mathbf{R}^2$$

$$\begin{aligned}
\text{s.t. } & \mathbf{x}(0) = \hat{\mathbf{x}}, & \\
& \mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j), & j = 0, \ldots, N_\mathsf{P} - 1 \\
& \mathbf{u}(-1) = \mathbf{u}_\mathsf{prev}, & \\
& \underline{\mathbf{x}} \le \mathbf{x}(j) \le \overline{\mathbf{x}}, & j = 1, \ldots, N_\mathsf{P} \\
& \underline{\mathbf{u}} \le \mathbf{u}(j) \le \overline{\mathbf{u}}, & j = 0, \ldots, N_\mathsf{P} - 1 \\
& |\mathbf{u}(j) - \mathbf{u}(j-1)| \le \boldsymbol{\Delta}_u, & j = 0, \ldots, N_\mathsf{P} - 1
\end{aligned}$$

KU LEUVEN

Why Model Predictive Control?

- Constraints incorporation
- Use of (approximate) process model: optimal solution for entire river system
- Prediction window + process model: rain predictions
- Objective function + constraints: set-point control together with flood control
- River systems have relatively slow dynamics

→ MPC is suitable for flood control of river systems

Outline

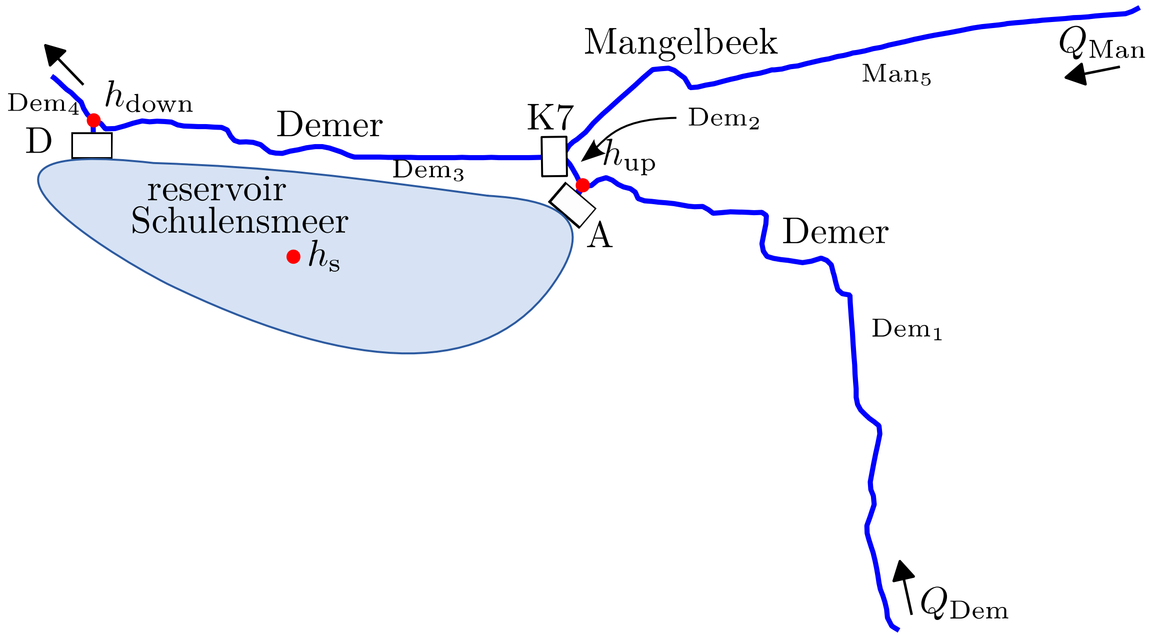
- Social relevance
- Modelling framework
- Model Predictive Control
- Conclusions

White box modelling

1. What do we need?
- Dynamics of a single reach
- Boundary conditions for connecting reaches
- Reservoirs

2. Numerical simulator

3. Approximate model



Dynamics of a single reach: The Saint-Venant equations

conservation of mass
$$\frac{\partial A}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial z} = 0$$

conservation of momentum
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial z}\frac{Q^2}{A} + gA\frac{\partial h}{\partial z} + gA(S_\mathsf{f} - S_0) = 0$$

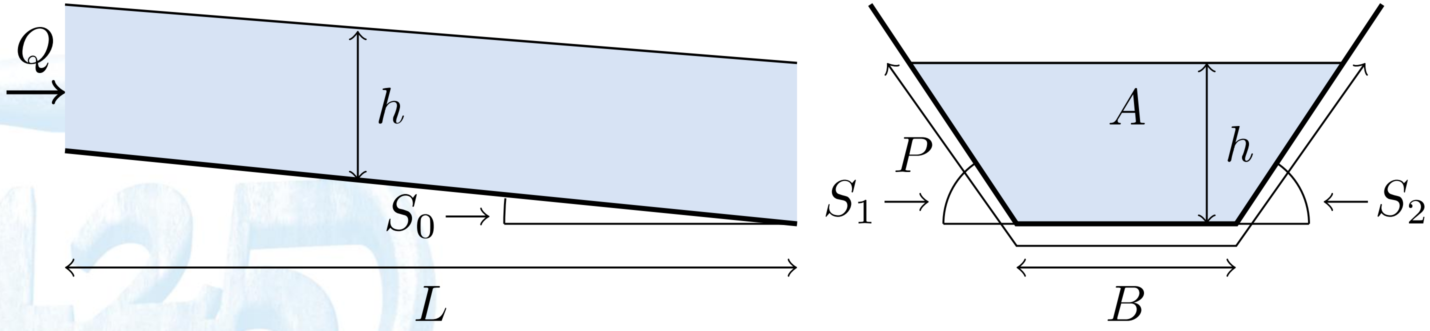
with

A the cross-sectional flow area (m^2)
Q water discharge (m^3/s)
h water depth (m)
S_0 bed slope
S_f friction slope

Dynamics of a single reach: The resistance law

The resistance law of Manning:

$$S_\mathsf{f} = n_\mathsf{mann}^2 \frac{Q|Q|}{A^2 R^{1/3}}$$

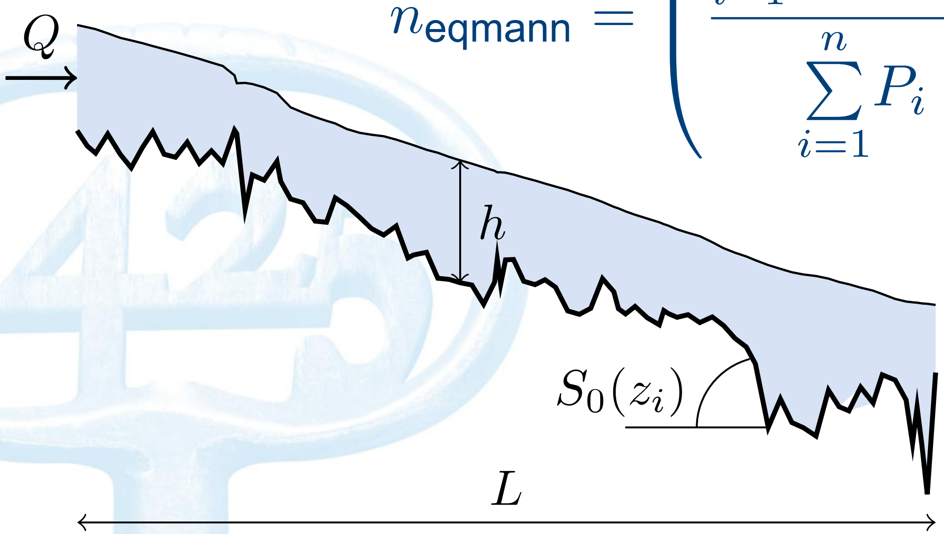


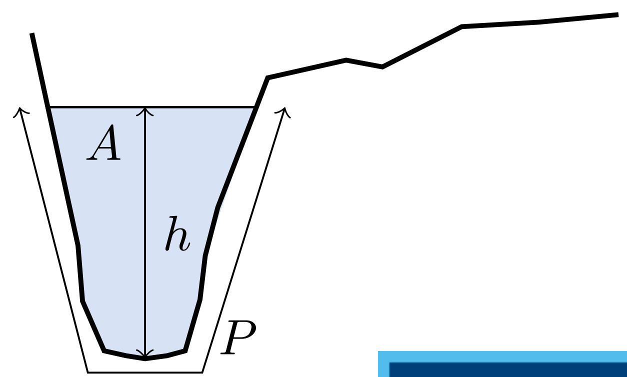
Dynamics of a single reach: The resistance law

The resistance law of Manning:

$$S_{\mathsf{f}} = n^2_{\mathsf{eqmann}} \frac{Q|Q|}{A^2 R^{1/3}}$$

$$n_{\mathsf{eqmann}} = \left(\frac{\sum_{i=1}^{n} P_i n^{3/2}_{\mathsf{mann},i}}{\sum_{i=1}^{n} P_i} \right)^{2/3}$$





Boundary conditions for a single reach

- Given upstream/downstream discharge

$$Q^{(i)}(0,t) = Q_{\mathsf{up}}(t)$$
$$Q^{(i)}(L^{(i)},t) = Q_{\mathsf{down}}(t)$$

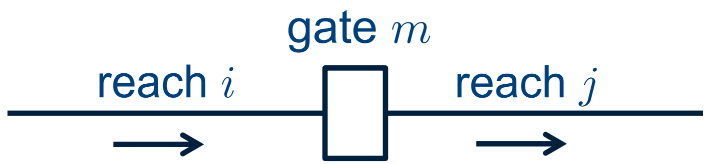
- Rating curve

$$f\left(h^{(i)}(L^{(i)},t), Q^{(i)}(L^{(i)},t)\right) = 0$$

Boundary conditions connecting reaches

- Hydraulic structures:

$$Q_{\text{gate}}(t) = \tilde{f}\left(c(t), h_{\text{up}}(t), h_{\text{down}}(t)\right)$$



$$Q^{(i)}(L^{(i)}, t) = Q^{(m)}_{\text{gate}}(t),$$

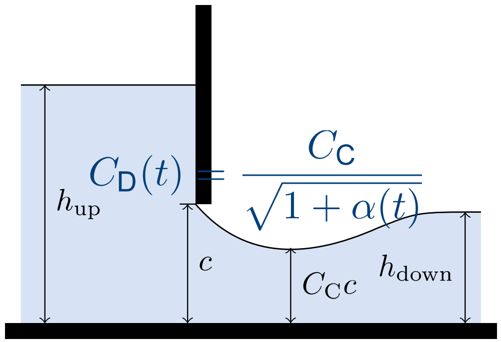
$$Q^{(j)}(0, t) = Q^{(m)}_{\text{gate}}(t),$$

$$Q^{(m)}_{\text{gate}}(t) = \tilde{f}\Big(c^{(m)}(t), h^{(i)}(L^{(i)}, t), h^{(j)}(0, t)\Big)$$

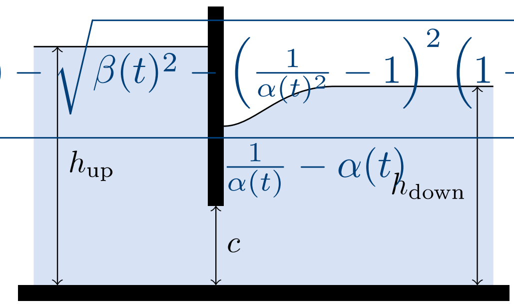
  - Vertical sluice
  - Gated weir

Boundary conditions connecting reaches

- Vertical sluice: $Q_{\text{gate}} = C_{\text{D}}(t) w c(t) \sqrt{2 g h_{\text{up}}(t)}$



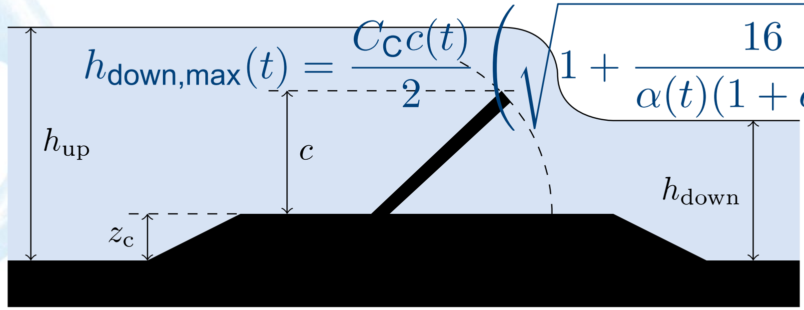
$$C_{\text{D}}(t) = \frac{C_{\text{C}}}{\sqrt{1+\alpha(t)}}$$



$$C_{\text{D}}(t) = C_{\text{C}} \left[\frac{\beta(t) - \sqrt{\beta(t)^2 - \left(\frac{1}{\alpha(t)^2} - 1\right)^2 \left(1 - \frac{1}{\gamma(t)^2}\right)}}{\frac{1}{\alpha(t)} - \alpha(t)} \right]^{1/2}$$

$$\beta(t) = (1/\alpha(t) - 1)^2 + 2\left(\gamma(t) - 1\right),$$
$$\alpha(t) = C_{\text{C}} c(t) / h_{\text{up}}(t),$$
$$\gamma(t) = h_{\text{up}}(t) / h_{\text{down}}(t),$$

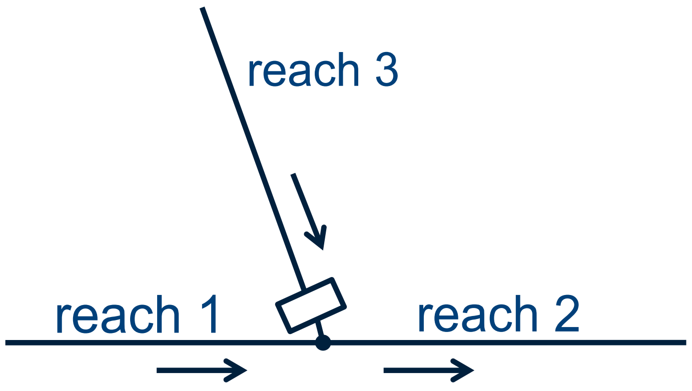
- Gated weir:



$$h_{\text{down,max}}(t) = \frac{C_{\text{C}} c(t)}{2} \left(\sqrt{1 + \frac{16}{\alpha(t)(1+\alpha(t))}} \right)$$

Boundary conditions connecting reaches

- Junctions



$$h^{(1)}(L^{(1)}, t) = h^{(2)}(0, t),$$

$$Q^{(1)}(L^{(1)}, t) + Q_{\text{gate}}(t) = Q^{(2)}(0, t),$$

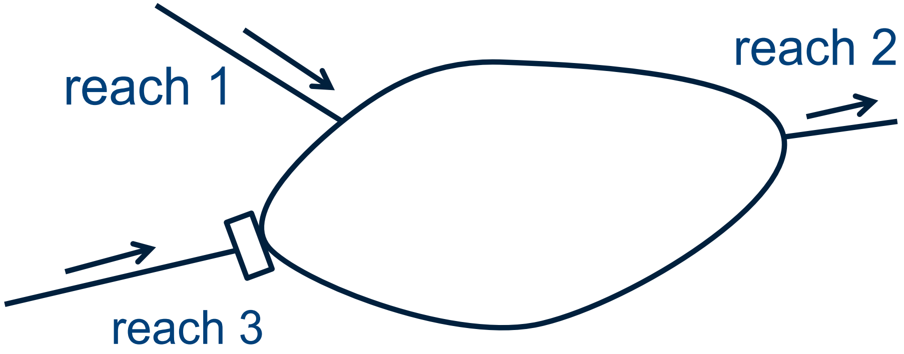
$$Q^{(3)}(L^{(3)}, t) = Q_{\text{gate}}(t),$$

$$Q_{\text{gate}}(t) = \tilde{f}\left(c^{(\text{gate})}(t), h^{(3)}(L^{(3)}, t), h^{(2)}(0, t)\right)$$

Reservoirs

Two options

- Saint-Venant equations



- Model as a tank

$$dV_{\text{res}}/dt = Q^{(1)}(L^{(1)}, t) + Q_{\text{gate}}(t) - Q^{(2)}(0, t)$$

KU LEUVEN

The hydrodynamic model of the Demer

For Dem$_1$, Dem$_2$, Dem$_3$, Dem$_4$ and Man$_5$:

$$\frac{\partial A}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial z} = 0$$

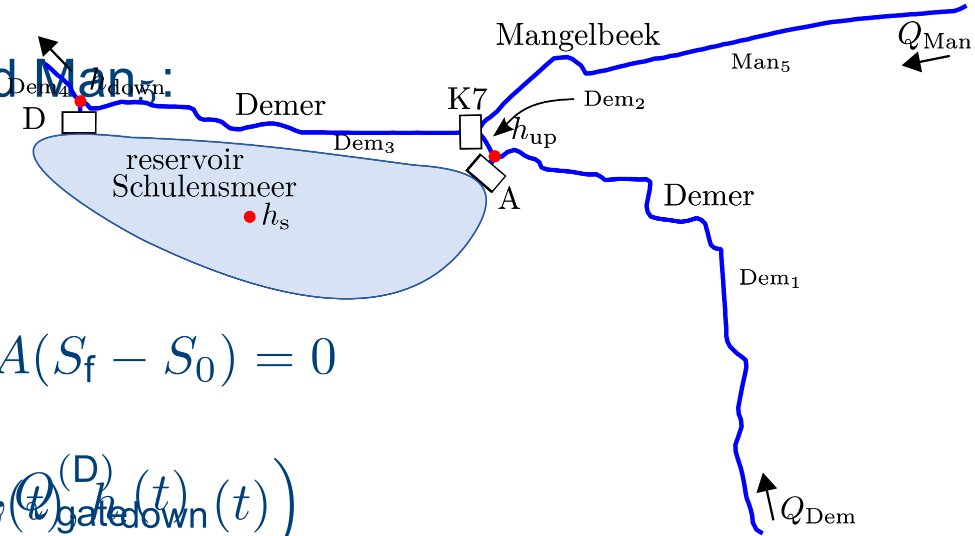
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial z}\frac{Q^2}{A} + gA\frac{\partial h}{\partial z} + gA(S_\mathsf{f} - S_0) = 0$$

with

$$S_\mathsf{f} = n^2_\mathsf{eqmann}\frac{Q|Q|}{A^2R^{4/3}}$$

with

$$n_\mathsf{eqmann} = \left(\frac{\sum_{i=1}^{n} P_i n^{3/2}_{\mathsf{mann},i}}{\sum_{i=1}^{n} P_i}\right)^{2/3}$$



For junction at gate A:

$$Q^{(5)}(L^{(5)},t) + Q^{(1)}(L^{(1)},t) = Q^{(2)}(0,t) + Q^{(\mathsf{A})}_\mathsf{gate}(t).$$

$$h^{(4)}(L^{(4)},t) = 0.9722 \cdot Q(L^{(4)},t)^2.$$

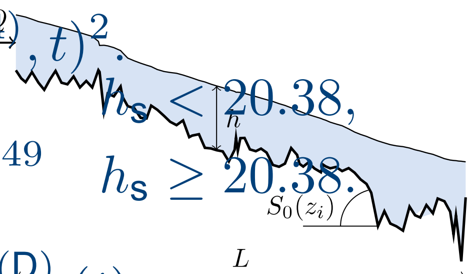
$$Q_\mathsf{gate}(t) = f\left(c(t), h_\mathsf{up}(t), h_\mathsf{down}(t)\right)$$

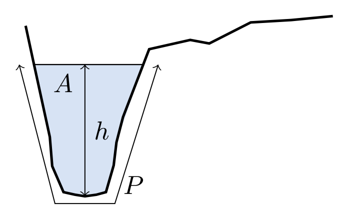
$$V_\mathsf{s} = \begin{cases} (h_\mathsf{s} - 20.38)/0.000771, & h_\mathsf{s} < 20.38, \\ ((h_\mathsf{s} - 20.38)/0.0007)^{1/0.838549}, & h_\mathsf{s} \geq 20.38. \end{cases}$$

For junction at gate D:

$$h^{(3)}(L^{(3)},t) = h^{(4)}(0,t),$$

$$Q^{(4)}(0,t) = Q^{(3)}(L^{(3)},t) + Q^{(\mathsf{D})}_\mathsf{gate}(t).$$



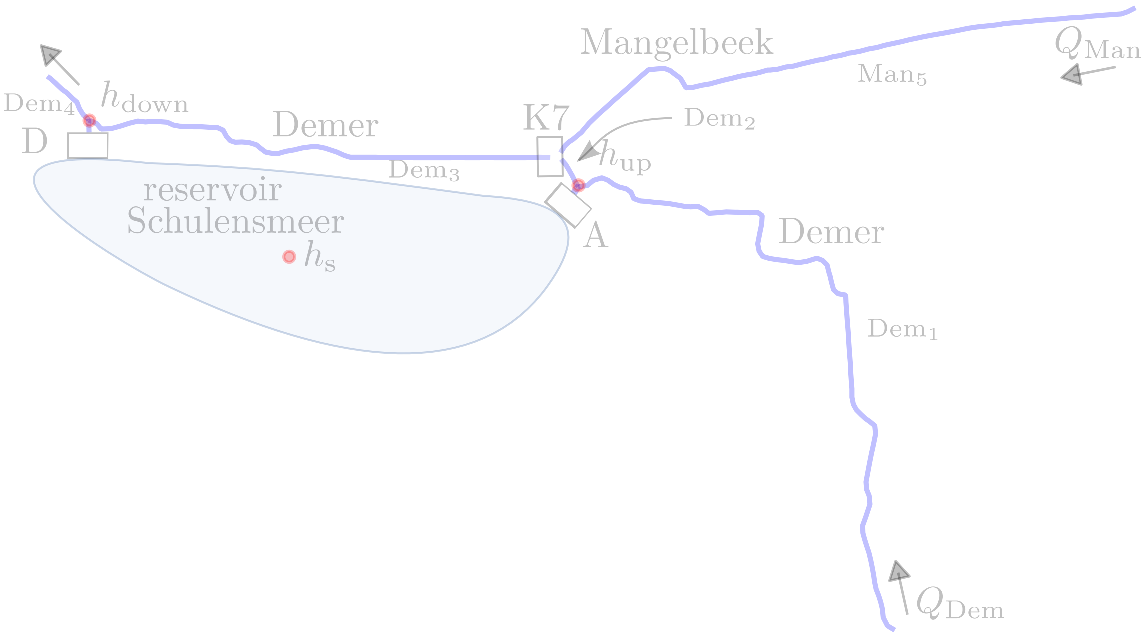


White box modelling

1. What do we need?
- Dynamics of a single reach
- Boundary conditions for connecting reaches
- Reservoirs

2. Numerical simulator

3. Approximate model



Numerical simulator

- For every reach:
- Approximate partial derivatives with finited differences

For PDE 1:

$$\frac{\partial A}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial z} = 0 \qquad \left(h^{(i)}(z_j, t_k) = h^{(i)}_{j,k}\right)$$

$$\frac{\partial h^{(i)}_{j,k}}{\partial t} \simeq \frac{h^{(i)}_{j,k+1} - h^{(i)}_{j,k}}{\Delta t},$$

$$\frac{\partial Q^{(i)}_{j,k}}{\partial z} \simeq \theta \frac{\left(Q^{(i)}_{j+1,k+1} - Q^{(i)}_{j,k+1}\right)}{\Delta z} + (1-\theta)\frac{\left(Q^{(i)}_{j+1,k} - Q^{(i)}_{j,k}\right)}{\Delta z},$$

$$\frac{\partial A^{(i)}_{j,k}}{\partial h} \simeq \theta \left(\frac{\partial A}{\partial h}\right)^{(i)}_{j,k+1} + (1-\theta)\left(\frac{\partial A}{\partial h}\right)^{(i)}_{j,k}$$

Numerical simulator

- For PDE 2:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial z}\frac{Q^2}{A} + gA\frac{\partial h}{\partial z} + gA(S_\mathrm{f} - S_0) = 0$$

$$\frac{\partial}{\partial z}\left(\frac{{Q^{(i)}}^2}{A^{(i)}}\right)_{j,k} \simeq \begin{cases} \frac{1}{\Delta z}\left(\left(\frac{{Q^{(i)}}^2}{A^{(i)}}\right)_{j+1,k+\theta} - \left(\frac{{Q^{(i)}}^2}{A^{(i)}}\right)_{j,k+\theta}\right) & Q^{(i)}_{j,k} < 0, \\ \frac{1}{\Delta z}\left(\left(\frac{{Q^{(i)}}^2}{A^{(i)}}\right)_{j,k+\theta} - \left(\frac{{Q^{(i)}}^2}{A^{(i)}}\right)_{j-1,k+\theta}\right) & Q^{(i)}_{j,k} \geq 0. \end{cases}$$

$$\Rightarrow \quad \mathbf{f}\left(\mathbf{h}^{(i)}(t_{k+1}), \mathbf{h}^{(i)}(t_k), \mathbf{q}^{(i)}(t_{k+1}), \mathbf{q}^{(i)}(t_k)\right) = \mathbf{0}_{n_h^{(i)}+n_Q^{(i)}-2}$$

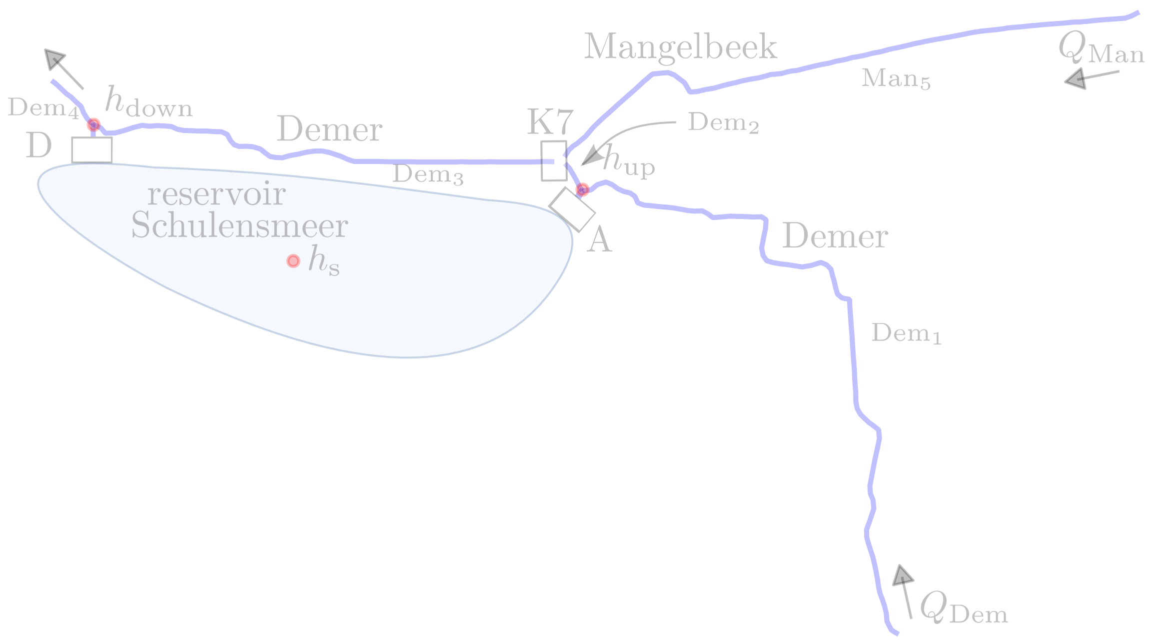
Use similar procedure for boundary conditions…

White box modelling

1. What do we need?
- Dynamics of a single reach
- Boundary conditions for connecting reaches
- Reservoirs

2. Numerical simulator

3. Approximate model



KU LEUVEN

Approximate model

- Goal: find an approximate model that is accurate enough but with a low complexity

- Linear state space model:

$$\mathbf{x}(k+1) = \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) + \tilde{\mathbf{E}}\mathbf{d}(k) + \tilde{\beta}$$

with

$$\mathbf{x}(k) = [\mathbf{h}(k); \mathbf{q}(k)],$$

$$\mathbf{u}(k) = \left[c^{(A)}(k); c^{(D)}(k); c^{(K7)}(k)\right],$$

$$\mathbf{d}(k) = [Q_{\text{Dem}}(k); Q_{\text{Man}}(k)]$$

Not accurate

Approximate model

- Linear-Nonlinear model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{F}\mathbf{d}(k) + \boldsymbol{\beta}$$

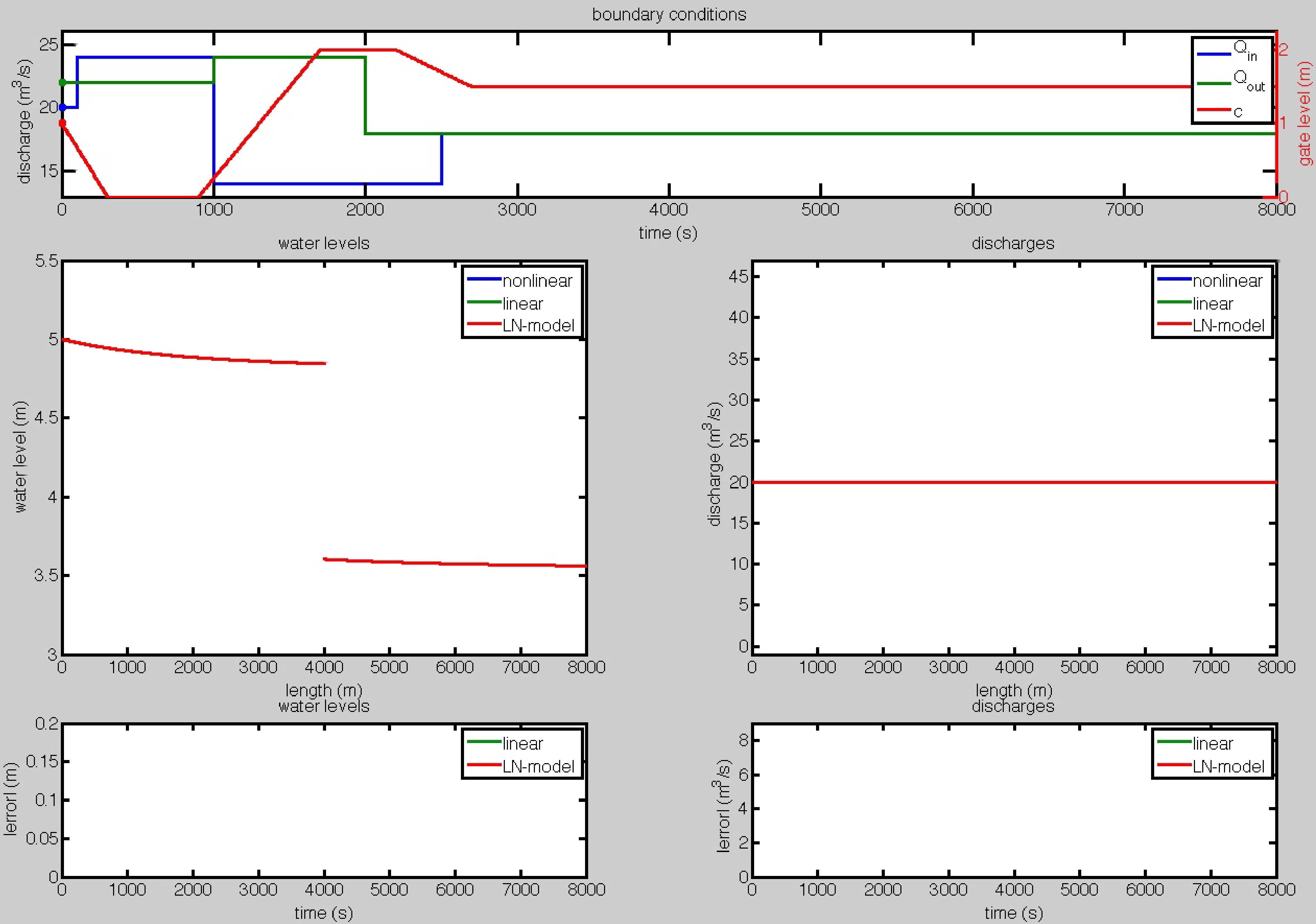
$$Q_{\text{gate}}^{(m)}(k) = \tilde{f}\left(c^{(m)}(k), h_{\text{up}}^{(m)}(k), h_{\text{down}}^{(m)}(k)\right), \text{ for } m = A, D, K7$$

with

$$\mathbf{x}(k) = [\mathbf{h}(k); \mathbf{q}(k)],$$

$$\mathbf{u}(k) = \left[Q_{\text{gate}}^{(A)}(k); Q_{\text{gate}}^{(D)}(k); Q_{\text{gate}}^{(K7)}(k)\right],$$

$$\mathbf{d}(k) = [Q_{\text{Dem}}(k); Q_{\text{Man}}(k)]$$

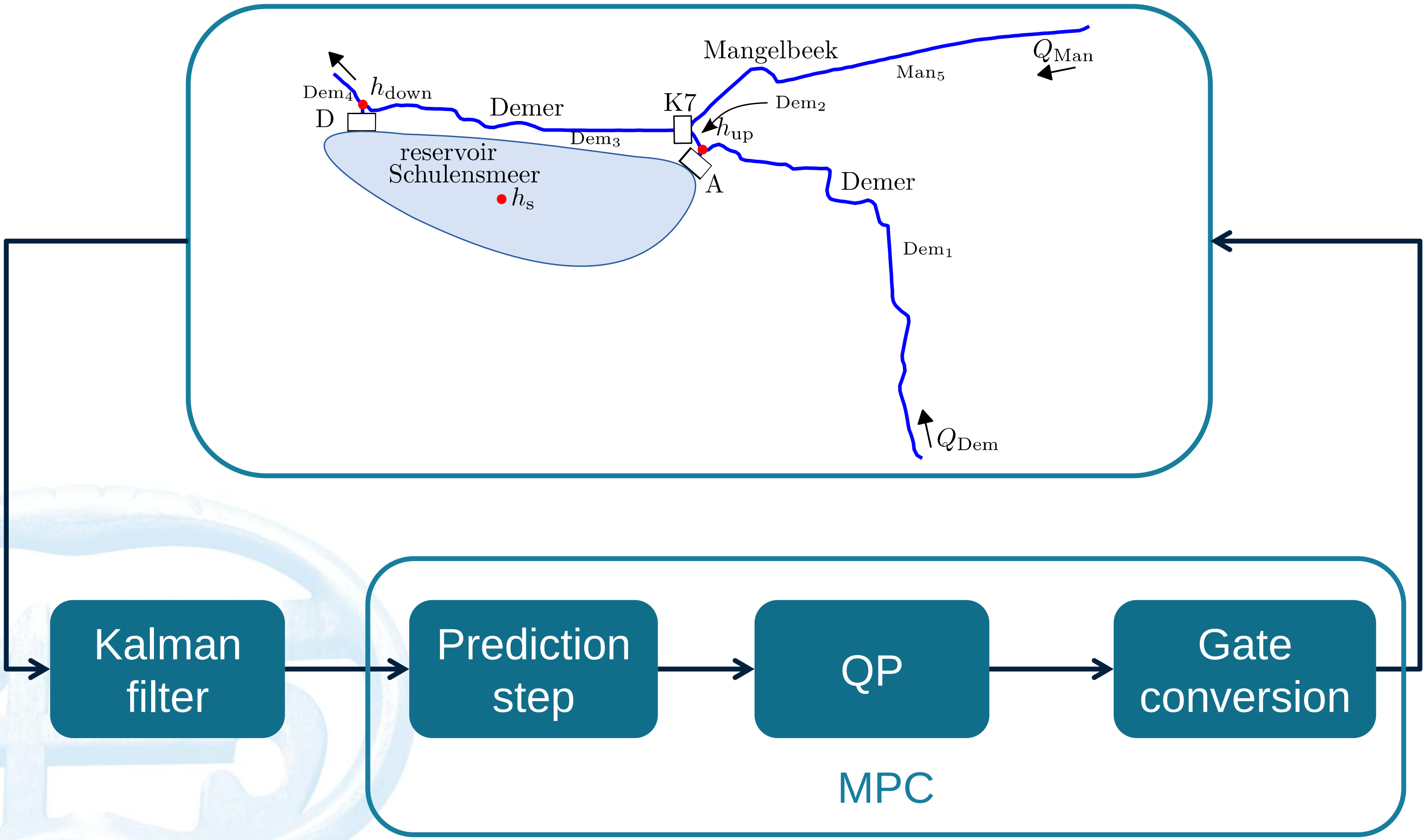


boundary conditions
discharge (m3/s)
gate level (m)
time (s)
Qin
Qout
c
water levels
water level (m)
length (m)
nonlinear
linear
LN-model
discharges
discharge (m3/s)
length (m)
nonlinear
linear
LN-model
water levels
|error| (m)
time (s)
linear
LN-model
discharges
|error| (m3/s)
time (s)
linear
LN-model

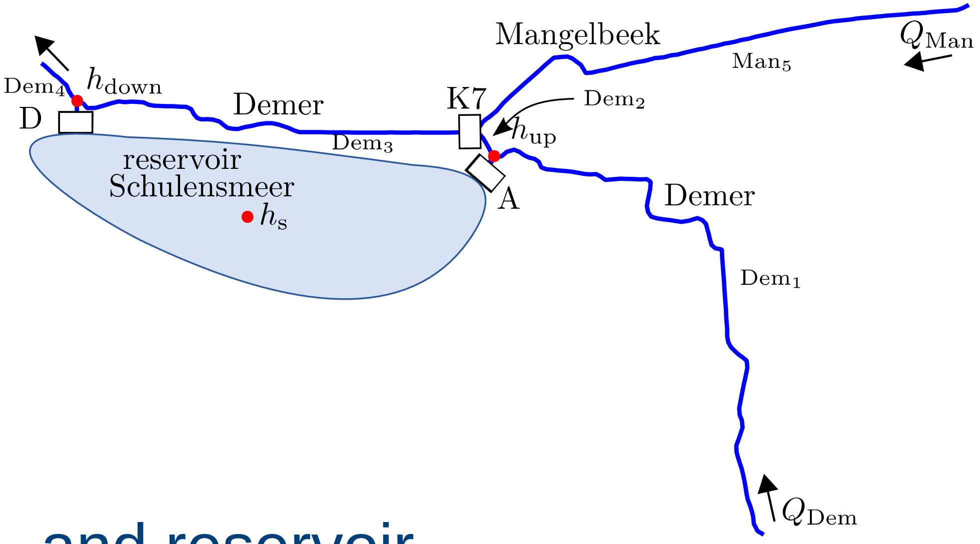
Outline

- Social relevance
- Modelling framework
- Model Predictive Control
- Conclusions

Model Predictive Control



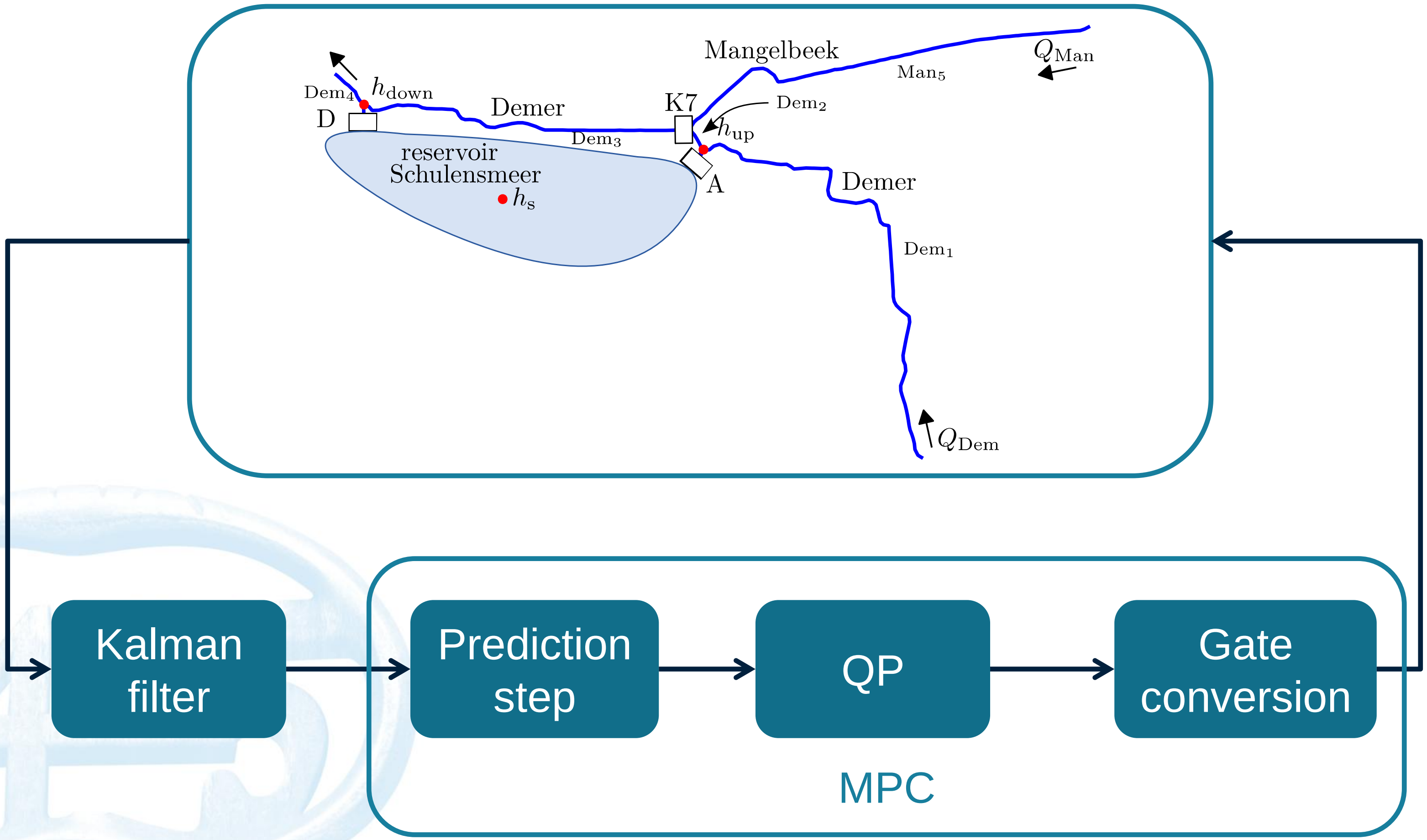
The requirements



- Control objectives:
  - Set-point control for h_{up} and reservoir
  - Flood control + respect safety limits and flood limits
  - Recovery of used buffer capacity

- Limitations:
  - Physical limits for gate positions: $\underline{\mathbf{c}}$, $\overline{\mathbf{c}}$, $\boldsymbol{\Delta}_c$
  - Only h_{up}, h_{s} and h_{down} are measured

Model Predictive Control



Model Predictive Control: Approximate model

Use LN-model

$$\mathbf{x}(k+1) = \mathbf{Ax}(k) + \mathbf{B}\begin{pmatrix} Q_{\text{gate}}^{(A)}(k) \\ Q_{\text{gate}}^{(D)}(k) \\ Q_{\text{gate}}^{(K7)}(k) \end{pmatrix} + \mathbf{Dd}(k) + \boldsymbol{\beta}$$

$$\begin{cases} Q_{\text{gate}}^{(A)}(k) = \tilde{f}\left(c^{(A)}(k), h^{(1)}(L^{(1),t_k}, h_{\text{s}}(t_k))\right) \\ Q_{\text{gate}}^{(D)}(k) = \tilde{f}\left(c^{(D)}(k), h_{\text{s}}(t_k), h^{(4)}(0, t_k)\right) \\ Q_{\text{gate}}^{(K7)}(k) = \tilde{f}\left(c^{(K7)}(k), h^{(2)}(L^{(2)}, t_k), h^{(3)}(0, t_k)\right) \end{cases}$$

but work only with linear part inside the optimization problem!
➔ optimize over gate discharges

KU LEUVEN

Model Predictive Control: The optimization problem

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\boldsymbol{\zeta}} \sum_{j=1}^{N_{\mathsf{P}}} \|\mathbf{x}(j)-\mathbf{r}_x\|_{\mathbf{W}}^2 + \sum_{j=0}^{N_{\mathsf{P}}-1} \|\mathbf{u}(j)-\mathbf{u}(j-1)\|_{\mathbf{R}}^2 + \\ + \sum_{j=0}^{N_{\mathsf{P}}-1} \|\mathbf{u}(j)-\mathbf{r}_u\|_{\mathbf{U}}^2 + \|\boldsymbol{\xi}\|_{\mathbf{S}}^2 + \mathbf{s}^{\mathsf{T}}\boldsymbol{\xi} + \|\boldsymbol{\zeta}\|_{\mathbf{V}}^2 + \mathbf{v}^{\mathsf{T}}\boldsymbol{\zeta}$$

$$\begin{aligned}
\text{s.t. } & \mathbf{x}(0) = \hat{\mathbf{x}}, \\
& \mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\boldsymbol{\beta}}(j), && j = 0,\dots,N_{\mathsf{P}}-1 \\
& \underline{\mathbf{u}}(j) \le \mathbf{u}(j) \le \overline{\mathbf{u}}(j), && j = 0,\dots,N_{\mathsf{P}}-1 \\
& \mathbf{u}(-1) = \mathbf{u}_{\mathsf{prev}}, \\
& \text{for } i = 1,\dots,5: \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},1} + \mathbf{1}_{n^{(i)}_{\mathsf{con}}} \cdot \eta(j)\xi_i, && j = 1,\dots,N_{\mathsf{P}} \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},2} + \mathbf{1}_{n^{(i)}_{\mathsf{con}}} \cdot \eta(j)\zeta_i, && j = 1,\dots,N_{\mathsf{P}} \\
& h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},1} + \eta(j)\xi_6, && j = 1,\dots,N_{\mathsf{P}} \\
& h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},2} + \eta(j)\zeta_6, && j = 1,\dots,N_{\mathsf{P}} \\
& \boldsymbol{\xi} \ge 0, \\
& \boldsymbol{\zeta} \ge 0
\end{aligned}$$

KU LEUVEN

Model Predictive Control: Flood control and set-point control

$$\min_{\mathbf{u},\mathbf{x},\xi,\zeta} \sum_{j=1}^{N_\mathsf{P}} \|\mathbf{x}(j) - \mathbf{r}_x\|_\mathbf{W}^2 + \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j) - \mathbf{u}(j-1)\|_\mathbf{R}^2 +$$

$$+ \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j) - \mathbf{r}_u\|_\mathbf{U}^2 + \|\xi\|_\mathbf{S}^2 + \mathbf{s}^\mathsf{T}\xi + \|\zeta\|_\mathbf{V}^2 + \mathbf{v}^\mathsf{T}\zeta$$

$$\begin{aligned}
\text{s.t. } & \mathbf{x}(0) = \hat{\mathbf{x}}, & \\
& \mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\beta}(j), & j = 0, \ldots, N_\mathsf{P} - 1 \\
& \underline{\mathbf{u}}(j) \le \mathbf{u}(j) \le \overline{\mathbf{u}}(j), & j = 0, \ldots, N_\mathsf{P} - 1 \\
& \mathbf{u}(-1) = \mathbf{u}_\mathsf{prev}, & \\
& \text{for } i = 1, \ldots, 5: & \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},1} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\xi_i, & j = 1, \ldots, N_\mathsf{P} \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},2} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\zeta_i, & j = 1, \ldots, N_\mathsf{P} \\
& h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},1} + \eta(j)\xi_6, & j = 1, \ldots, N_\mathsf{P} \\
& h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},2} + \eta(j)\zeta_6, & j = 1, \ldots, N_\mathsf{P} \\
& \xi \ge 0, & \\
& \zeta \ge 0 &
\end{aligned}$$

Model Predictive Control: Ensure feasibility of QP

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\zeta} \sum_{j=1}^{N_\mathrm{P}} \|\mathbf{x}(j) - \mathbf{r}_x\|_\mathbf{W}^2 + \sum_{j=0}^{N_\mathrm{P}-1} \|\mathbf{u}(j) - \mathbf{u}(j-1)\|_\mathbf{R}^2 +$$

$$+ \sum_{j=0}^{N_\mathrm{P}-1} \|\mathbf{u}(j) - \mathbf{r}_u\|_\mathbf{U}^2 + \|\boldsymbol{\xi}\|_\mathbf{S}^2 + \mathbf{s}^\mathsf{T}\boldsymbol{\xi} + \|\zeta\|_\mathbf{V}^2 + \mathbf{v}^\mathsf{T}\zeta$$

$$\text{s.t. } \mathbf{x}(0) = \hat{\mathbf{x}},$$

$$\mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\beta}(j), \qquad j = 0, \dots, N_\mathrm{P} - 1$$

$$\underline{\mathbf{u}}(j) \leq \mathbf{u}(j) \leq \overline{\mathbf{u}}(j), \qquad j = 0, \dots, N_\mathrm{P} - 1$$

$$\mathbf{u}(-1) = \mathbf{u}_\mathrm{prev},$$

$$\text{for } i = 1, \dots, 5:$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \leq \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathrm{max},1} + \mathbf{1}_{n^{(i)}_\mathrm{con}} \cdot \eta(j)\xi_i, \qquad j = 1, \dots, N_\mathrm{P}$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \leq \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathrm{max},2} + \mathbf{1}_{n^{(i)}_\mathrm{con}} \cdot \eta(j)\zeta_i, \qquad j = 1, \dots, N_\mathrm{P}$$

$$h^{(\mathrm{schulen})}(j) \leq h^{(\mathrm{schulen})}_{\mathrm{max},1} + \eta(j)\xi_6, \qquad j = 1, \dots, N_\mathrm{P}$$

$$h^{(\mathrm{schulen})}(j) \leq h^{(\mathrm{schulen})}_{\mathrm{max},2} + \eta(j)\zeta_6, \qquad j = 1, \dots, N_\mathrm{P}$$

$$\boldsymbol{\xi} \geq 0,$$

$$\zeta \geq 0$$

Model Predictive Control: Control objectives ➔ weighting matrices

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\boldsymbol{\zeta}} \sum_{j=1}^{N_\mathrm{P}} \|\mathbf{x}(j)-\mathbf{r}_x\|^2_{\mathbf{W}} + \sum_{j=0}^{N_\mathrm{P}-1} \|\mathbf{u}(j)-\mathbf{u}(j-1)\|^2_{\mathbf{R}} +$$

$$+\sum_{j=0}^{N_\mathrm{P}-1} \|\mathbf{u}(j)-\mathbf{r}_u\|^2_{\mathbf{U}} + \|\boldsymbol{\xi}\|^2_{\mathbf{S}} + \mathbf{s}^\mathsf{T}\boldsymbol{\xi} + \|\boldsymbol{\zeta}\|^2_{\mathbf{V}} + \mathbf{v}^\mathsf{T}\boldsymbol{\zeta}$$

s.t. $\mathbf{x}(0) = \hat{\mathbf{x}},$

$$\mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\beta}(j), \qquad j = 0,\dots,N_\mathrm{P}-1$$

$$\underline{\mathbf{u}}(j) \le \mathbf{u}(j) \le \overline{\mathbf{u}}(j), \qquad j = 0,\dots,N_\mathrm{P}-1$$

$$\mathbf{u}(-1) = \mathbf{u}_\mathrm{prev},$$

for $i = 1,\dots,5:$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathrm{max},1} + \mathbf{1}_{n^{(i)}_\mathrm{con}} \cdot \eta(j)\xi_i, \qquad j = 1,\dots,N_\mathrm{P}$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathrm{max},2} + \mathbf{1}_{n^{(i)}_\mathrm{con}} \cdot \eta(j)\zeta_i, \qquad j = 1,\dots,N_\mathrm{P}$$

$$h^{(\mathrm{schulen})}(j) \le h^{(\mathrm{schulen})}_{\mathrm{max},1} + \eta(j)\xi_6, \qquad j = 1,\dots,N_\mathrm{P}$$

$$h^{(\mathrm{schulen})}(j) \le h^{(\mathrm{schulen})}_{\mathrm{max},2} + \eta(j)\zeta_6, \qquad j = 1,\dots,N_\mathrm{P}$$

$$\boldsymbol{\xi} \ge 0,$$

$$\boldsymbol{\zeta} \ge 0$$

KU LEUVEN

Model Predictive Control: Limits on gate discharges & model update

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\boldsymbol{\zeta}} \sum_{j=1}^{N_\mathsf{P}} \|\mathbf{x}(j)-\mathbf{r}_x\|^2_{\mathbf{W}} + \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j)-\mathbf{u}(j-1)\|^2_{\mathbf{R}} +$$

$$+ \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j)-\mathbf{r}_u\|^2_{\mathbf{U}} + \|\boldsymbol{\xi}\|^2_{\mathbf{S}} + \mathbf{s}^\mathsf{T}\boldsymbol{\xi} + \|\boldsymbol{\zeta}\|^2_{\mathbf{V}} + \mathbf{v}^\mathsf{T}\boldsymbol{\zeta}$$

$$\text{s.t. } \mathbf{x}(0) = \hat{\mathbf{x}},$$

$$\mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\boldsymbol{\beta}}(j), \qquad j = 0,\ldots,N_\mathsf{P}-1$$

$$\underline{\mathbf{u}}(j) \le \mathbf{u}(j) \le \overline{\mathbf{u}}(j), \qquad j = 0,\ldots,N_\mathsf{P}-1$$

$$\mathbf{u}(-1) = \mathbf{u}_\mathsf{prev},$$

$$\text{for } i = 1,\ldots,5:$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},1} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\xi_i, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},2} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\zeta_i, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},1} + \eta(j)\xi_6, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$h^{(\mathsf{schulen})}(j) \le h^{(\mathsf{schulen})}_{\mathsf{max},2} + \eta(j)\zeta_6, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$\boldsymbol{\xi} \ge 0,$$

$$\boldsymbol{\zeta} \ge 0$$

KU LEUVEN

Model Predictive Control: $\underline{\mathbf{c}}$, $\overline{\mathbf{c}}$, $\Delta_c \Rightarrow \underline{\mathbf{u}}(j)$, $\overline{\mathbf{u}}(j)$

At time t_k: $\mathbf{c}(t_{k-1})$, $\mathbf{h}(t_k)$ and $\mathbf{q}(t_k)$ are known.
For gate m:

$$\underline{u}^{(m)}(k) = \tilde{f}\left(c^{(m)}(k-1) + \Delta_c, h_{\text{up}}(k), h_{\text{down}}(k)\right),$$

$$\overline{u}^{(m)}(k) = \tilde{f}\left(c^{(m)}(k-1) - \Delta_c, h_{\text{up}}(k), h_{\text{down}}(k)\right).$$

For $\underline{u}^{(m)}(k+1)$, $\overline{u}^{(m)}(k+1)$?

- $\mathbf{h}(t_{k+1})$? use (non)linear model to predict $\mathbf{x}(k+1)$ based on $\mathbf{x}(k)$, $\mathbf{d}(k)$ and $\mathbf{u}_{\text{opt}}(k)$
- $\mathbf{c}(t_k)$? use $\mathbf{u}_{\text{opt}}(k)$ but prevent uncontrollability of gates!

KU LEUVEN

Model Predictive Control: Model update

- Update linear model to match predictions with nonlinear model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{D}\mathbf{d}(k) + \tilde{\boldsymbol{\beta}}(k)$$

with

$$\tilde{\boldsymbol{\beta}}(k) = \boldsymbol{\beta} + (\mathbf{x}_{\text{nonlin}}(k+1) - \mathbf{x}_{\text{lin}}(k+1))$$

Model Predictive Control: Buffer capacity recovery

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\boldsymbol{\zeta}} \sum_{j=1}^{N_\mathsf{P}} \|\mathbf{x}(j) - \mathbf{r}_x\|^2_{\mathbf{W}} + \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j) - \mathbf{u}(j-1)\|^2_{\mathbf{R}} +$$

$$+ \sum_{j=0}^{N_\mathsf{P}-1} \|\mathbf{u}(j) - \mathbf{r}_u\|^2_{\mathbf{U}} + \|\boldsymbol{\xi}\|^2_{\mathbf{S}} + \mathbf{s}^\mathsf{T}\boldsymbol{\xi} + \|\boldsymbol{\zeta}\|^2_{\mathbf{V}} + \mathbf{v}^\mathsf{T}\boldsymbol{\zeta}$$

$$\text{s.t. } \mathbf{x}(0) = \hat{\mathbf{x}},$$

$$\mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\boldsymbol{\beta}}(j), \qquad j = 0,\ldots,N_\mathsf{P}-1$$

$$\underline{\mathbf{u}}(j) \leq \mathbf{u}(j) \leq \overline{\mathbf{u}}(j), \qquad j = 0,\ldots,N_\mathsf{P}-1$$

$$\mathbf{u}(-1) = \mathbf{u}_\mathsf{prev},$$

$$\text{for } i = 1,\ldots,5:$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \leq \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},1} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\xi_i, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$\mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \leq \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\mathsf{max},2} + \mathbf{1}_{n^{(i)}_\mathsf{con}} \cdot \eta(j)\zeta_i, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$h^{(\mathsf{schulen})}(j) \leq h^{(\mathsf{schulen})}_{\mathsf{max},1} + \eta(j)\xi_6, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$h^{(\mathsf{schulen})}(j) \leq h^{(\mathsf{schulen})}_{\mathsf{max},2} + \eta(j)\zeta_6, \qquad j = 1,\ldots,N_\mathsf{P}$$

$$\boldsymbol{\xi} \geq 0,$$

$$\boldsymbol{\zeta} \geq 0$$

KU LEUVEN

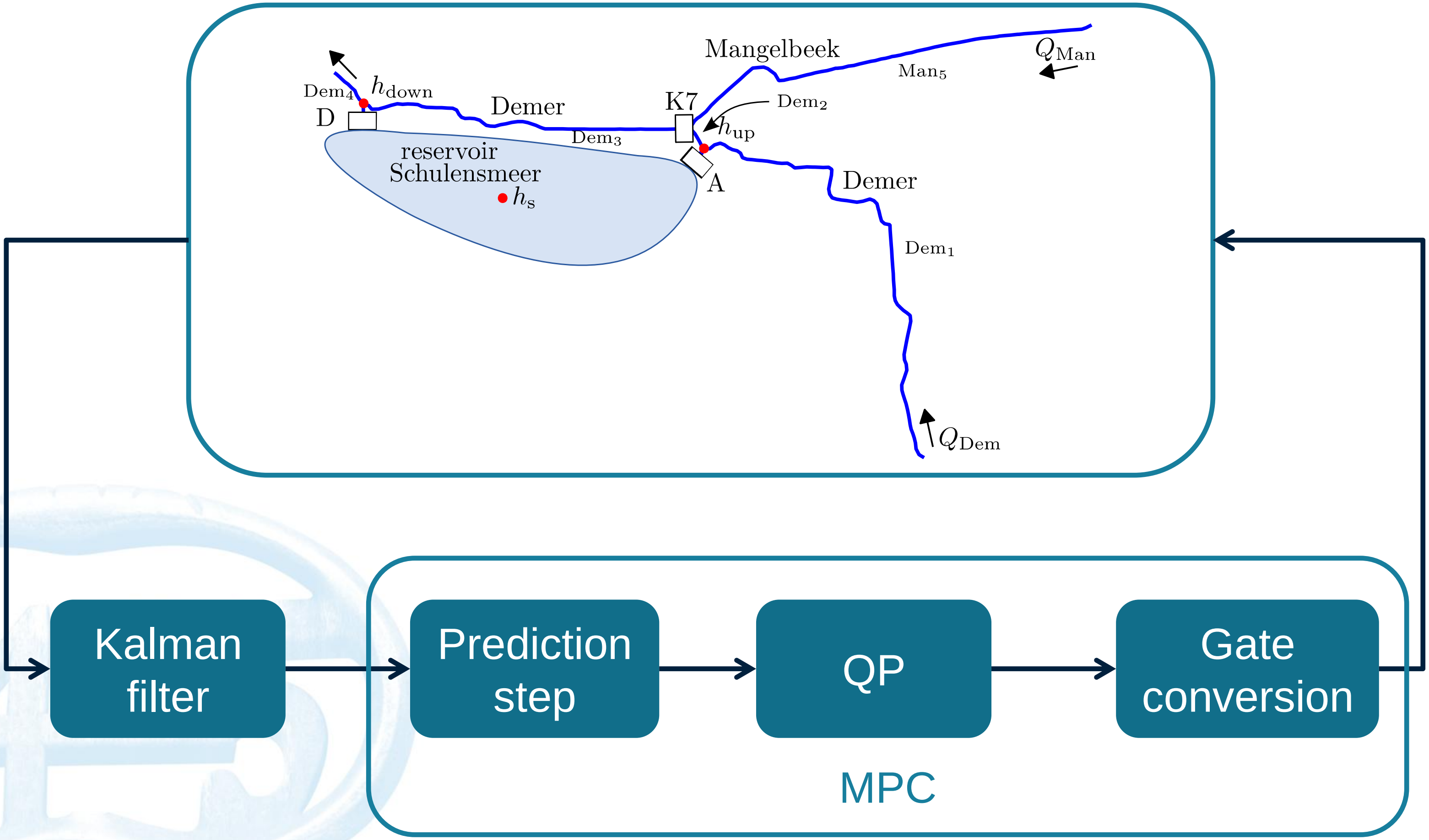
Model Predictive Control: Constraint selection

$$\min_{\mathbf{u},\mathbf{x},\boldsymbol{\xi},\boldsymbol{\zeta}} \sum_{j=1}^{N_\text{P}} \|\mathbf{x}(j)-\mathbf{r}_x\|_\mathbf{W}^2 + \sum_{j=0}^{N_\text{P}-1} \|\mathbf{u}(j)-\mathbf{u}(j-1)\|_\mathbf{R}^2 +$$

$$+ \sum_{j=0}^{N_\text{P}-1} \|\mathbf{u}(j)-\mathbf{r}_u\|_\mathbf{U}^2 + \|\boldsymbol{\xi}\|_\mathbf{S}^2 + \mathbf{s}^\mathsf{T}\boldsymbol{\xi} + \|\boldsymbol{\zeta}\|_\mathbf{V}^2 + \mathbf{v}^\mathsf{T}\boldsymbol{\zeta}$$

$$\begin{aligned}
\text{s.t. } & \mathbf{x}(0) = \hat{\mathbf{x}}, \\
& \mathbf{x}(j+1) = \mathbf{A}\mathbf{x}(j) + \mathbf{B}\mathbf{u}(j) + \mathbf{F}\mathbf{d}(j) + \tilde{\boldsymbol{\beta}}(j), && j = 0,\dots,N_\text{P}-1 \\
& \underline{\mathbf{u}}(j) \le \mathbf{u}(j) \le \overline{\mathbf{u}}(j), && j = 0,\dots,N_\text{P}-1 \\
& \mathbf{u}(-1) = \mathbf{u}_\text{prev}, \\
& \text{for } i = 1,\dots,5: \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\text{max},1} + \mathbf{1}_{n^{(i)}_\text{con}} \cdot \eta(j)\xi_i, && j = 1,\dots,N_\text{P} \\
& \quad \mathbf{M}^{(i)}\mathbf{h}^{(i)}(j) \le \mathbf{M}^{(i)}\mathbf{h}^{(i)}_{\text{max},2} + \mathbf{1}_{n^{(i)}_\text{con}} \cdot \eta(j)\zeta_i, && j = 1,\dots,N_\text{P} \\
& h^{(\text{schulen})}(j) \le h^{(\text{schulen})}_{\text{max},1} + \eta(j)\xi_6, && j = 1,\dots,N_\text{P} \\
& h^{(\text{schulen})}(j) \le h^{(\text{schulen})}_{\text{max},2} + \eta(j)\zeta_6, && j = 1,\dots,N_\text{P} \\
& \boldsymbol{\xi} \ge 0, \\
& \boldsymbol{\zeta} \ge 0
\end{aligned}$$

Model Predictive Control



Kalman Filter

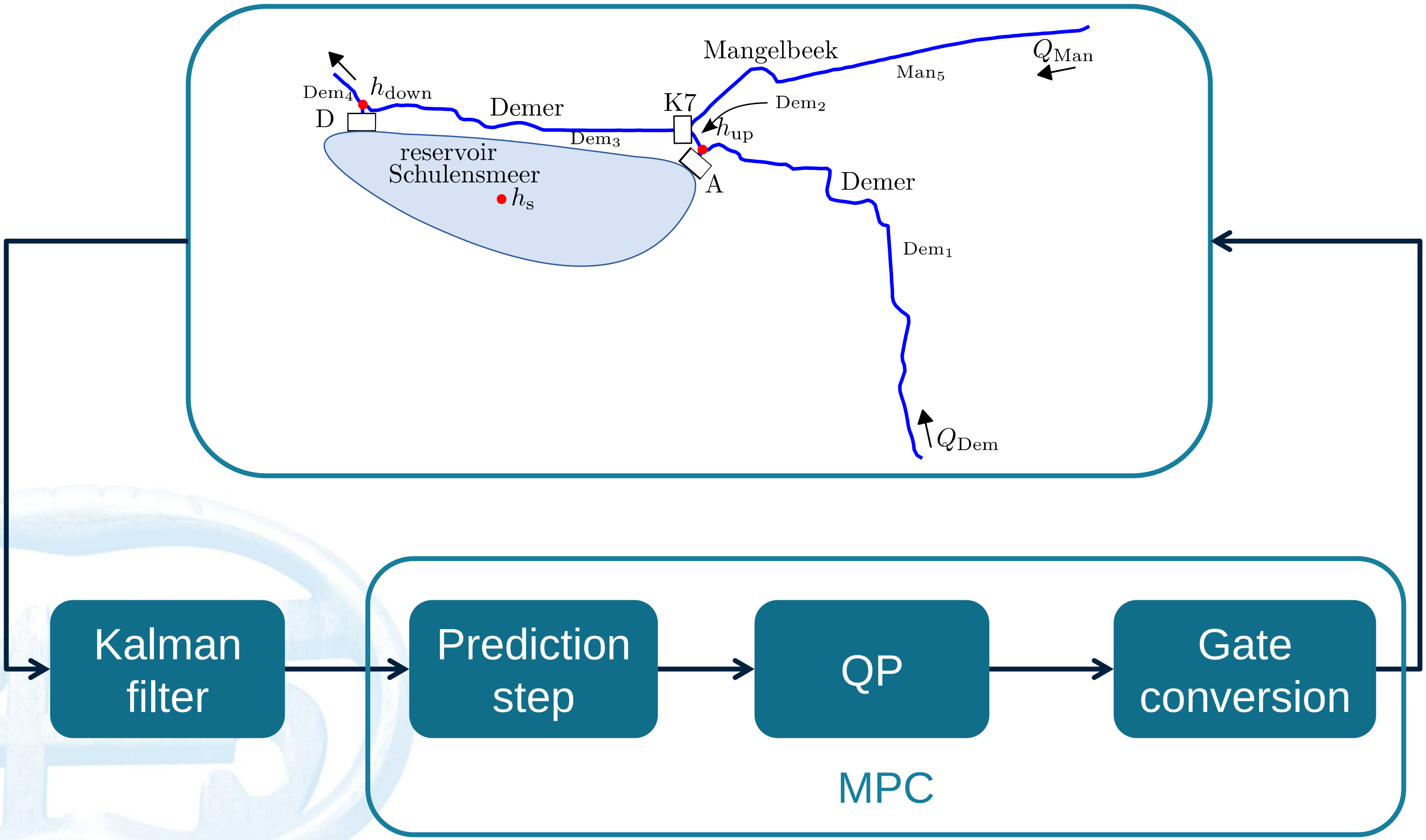
Estimate the entire state of the river system based on the three measured water levels together with the control actions:

$$\hat{\mathbf{x}}(k+1) = \mathbf{L}\left(\boldsymbol{\Delta}\mathbf{y}(k) - \boldsymbol{\Delta}\hat{\mathbf{y}}(k)\right) + \mathbf{x}_{\text{nonlin}}(k+1)$$

$$\Delta\hat{\mathbf{x}}(k+1) = \mathbf{L}\left(\Delta\mathbf{y}(k) - \Delta\hat{\mathbf{y}}(k)\right) + \mathbf{A}\Delta\hat{\mathbf{x}}\cancel{(k) + \mathbf{B}\Delta\mathbf{u}(k) +} \mathbf{D}\Delta\mathbf{d}(k),$$

$$\Delta\hat{\mathbf{y}}(k) = \mathbf{C}\Delta\hat{\mathbf{x}}(k)$$

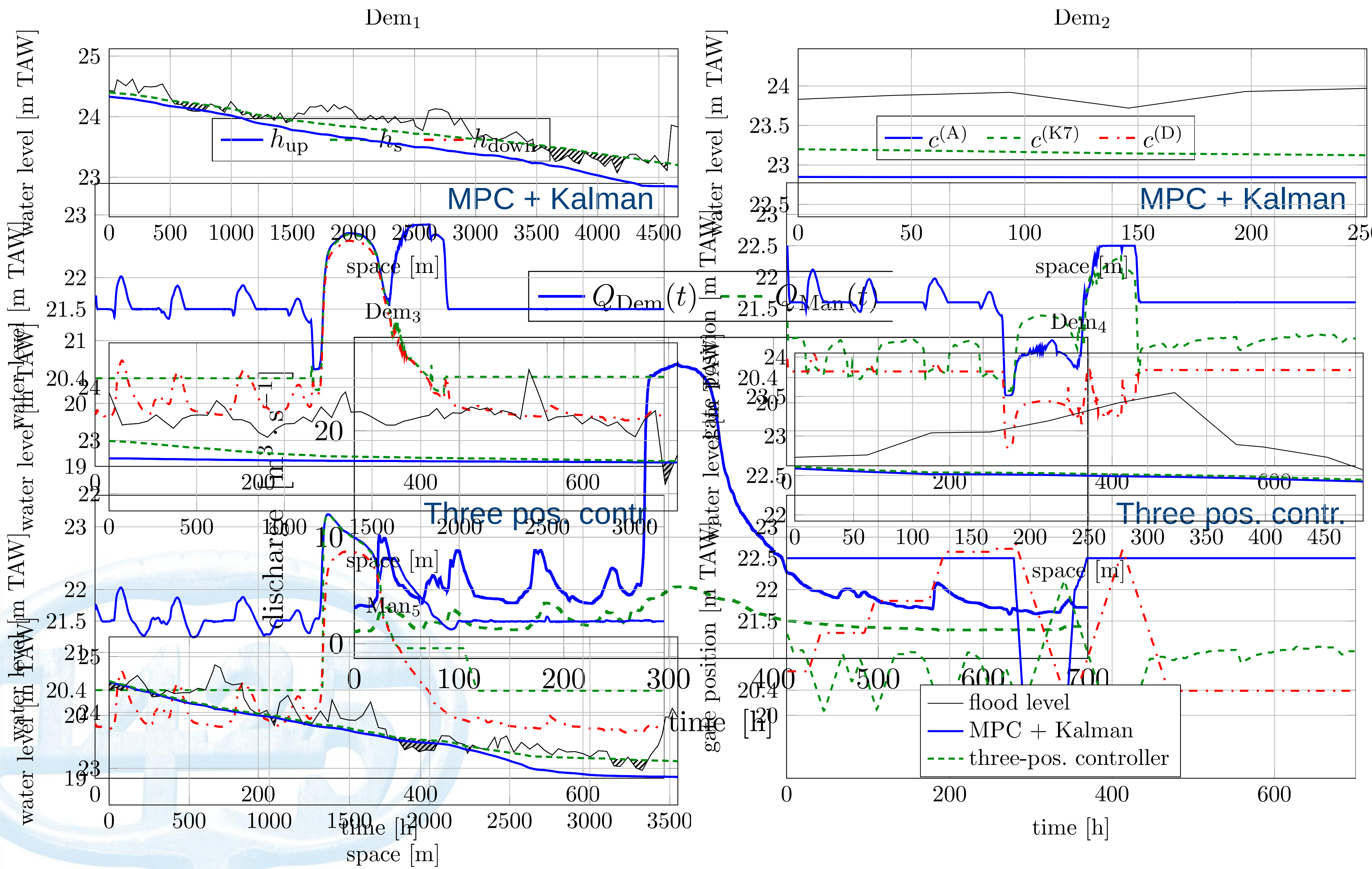
Model Predictive Control: The proof of the pudding



Simulation results

	Dem_1	Dem_2	Dem_3	Dem_4	Man_5	Schulensmeer
$\mathbf{W} \in \mathbb{R}^{534\times 534}$						
water levels	$[10^{-3}\cdot \mathbf{1}_{94}; 10]$ $[10^{-3}\cdot \mathbf{1}_{94}; 0.01]^{(*)}$	$10^{-3}\cdot \mathbf{1}_{6}$	$10^{-3}\cdot \mathbf{1}_{66}$ $0.01\cdot \mathbf{1}_{66}^{(*)}$	$10^{-3}\cdot \mathbf{1}_{11}$ $0.01\cdot \mathbf{1}_{11}^{(*)}$	$10^{-3}\cdot \mathbf{1}_{86}$	50
discharges	$10^{-3}\cdot \mathbf{1}_{96}$	$10^{-3}\cdot \mathbf{1}_{7}$	$10^{-3}\cdot \mathbf{1}_{67}$	$10^{-3}\cdot \mathbf{1}_{12}$	$10^{-3}\cdot \mathbf{1}_{87}$	
$\mathbf{S} \in \mathbb{R}^{6\times 6}$						
safety levels	10^3	10^3	10^3	10^3	10^3	10^4
$\mathbf{s} \in \mathbb{R}^{6\times 1}$						
safety levels	10^3	10^3	10^3	10^3	10^3	10^4
$\mathbf{V} \in \mathbb{R}^{6\times 6}$						
flood levels	10^5	10^5	10^5	10^5	10^5	10^5
$\mathbf{v} \in \mathbb{R}^{6\times 1}$						
flood levels	10^5	10^5	10^5	10^5	10^5	10^5

	$Q^{(\text{A})}$	$Q^{(\text{K7})}$	$Q^{(\text{D})}$
$\mathbf{R} \in \mathbb{R}^{3\times 3}$			
control actions	0.01	0.01	0.01
$\mathbf{U} \in \mathbb{R}^{3\times 3}$			
control actions	1000 $0.001^{(*)}$	0.001	1000 $0.001^{(*)}$



Dem1
Dem2
Dem3
Dem4
Man5
MPC + Kalman
Three pos. contr.
water level [m TAW]
space [m]
time [h]
discharge [m^3 s^-1]
gate position [m TAW]
flood level
MPC + Kalman
three-pos. controller

Simulation results

	Dem_1	Dem_2	Dem_3	Dem_4	Man_5	Schulensmeer
maximal flooding [m]						
three-pos. controller	0.275	-0.573	0.432	-0.119	0.216	-0.006
MPC+Kalman	0.036	-0.875	0.409	-0.142	0.168	-0.509
total flooding [m]						
three-pos. controller	2877	0	1243	0	4096	0
MPC+Kalman	69	0	1138	0	2032	0
flood duration [h]						
three-pos. controller	49.1	0	63.1	0	60.5	0
MPC+Kalman	27.3	0	62.9	0	46.6	0

Outline

- Social relevance
- Modelling framework
- Model Predictive Control
- Conclusions

Conclusions

Objective:
Can Model Predictive Control be used for
set-point control and flood control of river systems?

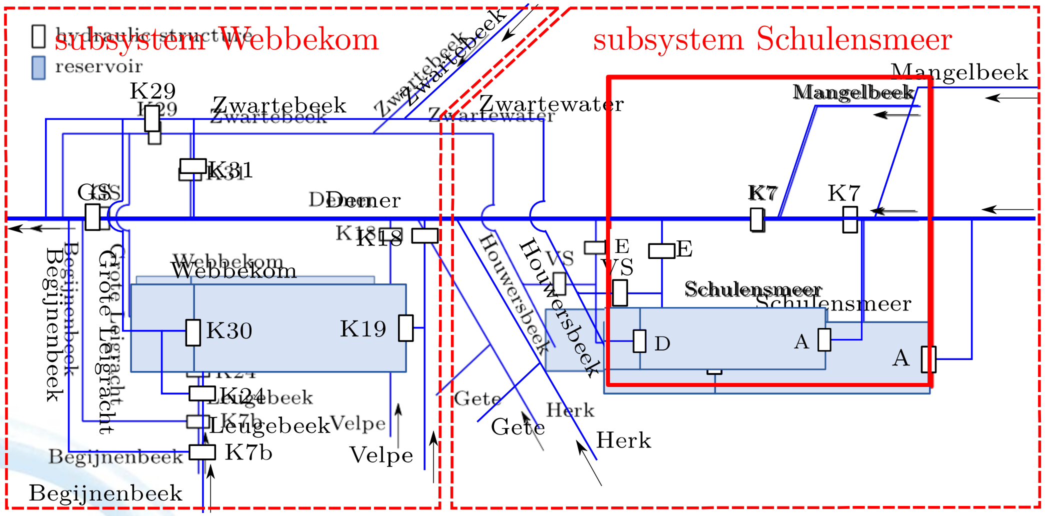
YES

Good control performance due to
- incorporation of flood levels as (soft) constraints
- minimization of the set-point deviations
- incorporation of rain predictions via process model and prediction window
- fast buffer capacity recovery

Important: smart choice of control variables ➔ linear MPC
Kalman filter as state estimator

Future research opportunities

- Apply to larger part of the Demer



Distributed MPC – Hierarchical MPC ?

- Plant-model mismatch
- Uncertainty on weather predictions

Thank you for your attention!

Flood control of river systems with Model Predictive Control

The river Demer as case study

Maarten Breckpot

Jury:
Y. Willems, chair
B. De Moor, promotor
P. Willems
M. Diehl
B. De Schutter
(TU Delft)
B. Pluymers
(IPCOS NV)

KU LEUVEN

Dynamics of a single reach: The Saint-Venant equations

Assumptions:

- The vertical pressure distribution is hydrostatic.
- The channel bottom slope is small: the flow depth measured normal to the channel bottom or measured vertically are approximately the same.
- The bedding of the channel is stable: the bed elevation does not change with time.
- The flow is assumed to be one-dimensional (flow velocity over the entire channel is uniform + water level across the section is horizontal).
- The frictional bed resistance is the same in unsteady flow as in steady flow meaning that steady state resistance laws can be used to evaluate the average boundary shear stress.

Numerical simulator: Δz, Δt, θ

- Numerical scheme is unconditional stable if

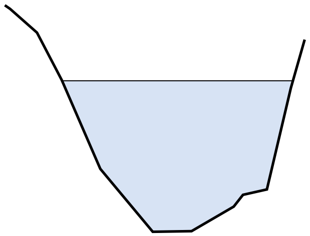
$$\theta \in \left[\frac{1}{2}, 1\right]$$

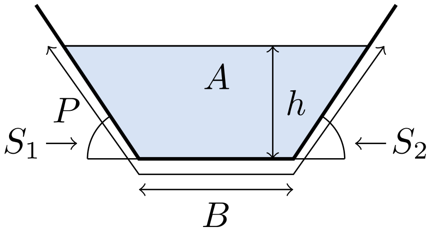
- Accuracy affected by Courant number

$$C_\mathsf{n} = \frac{|v| \pm \sigma}{\Delta z / \Delta t}$$

Adaptations to MPC scheme: Approximate model

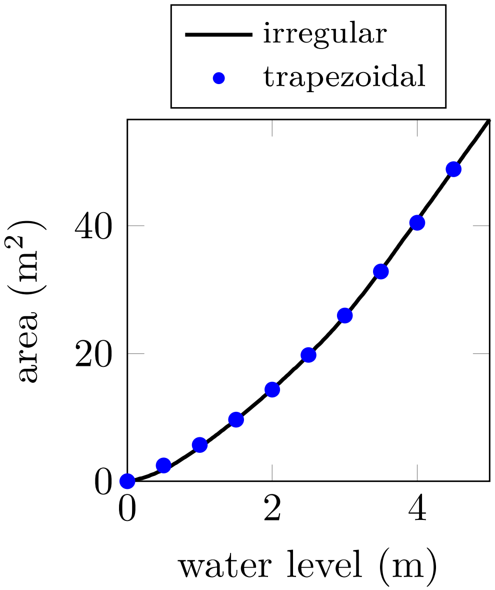
- Use (linear part of) LN-model …
  but first approximate the irregular profiles with trapezoidal cross sections





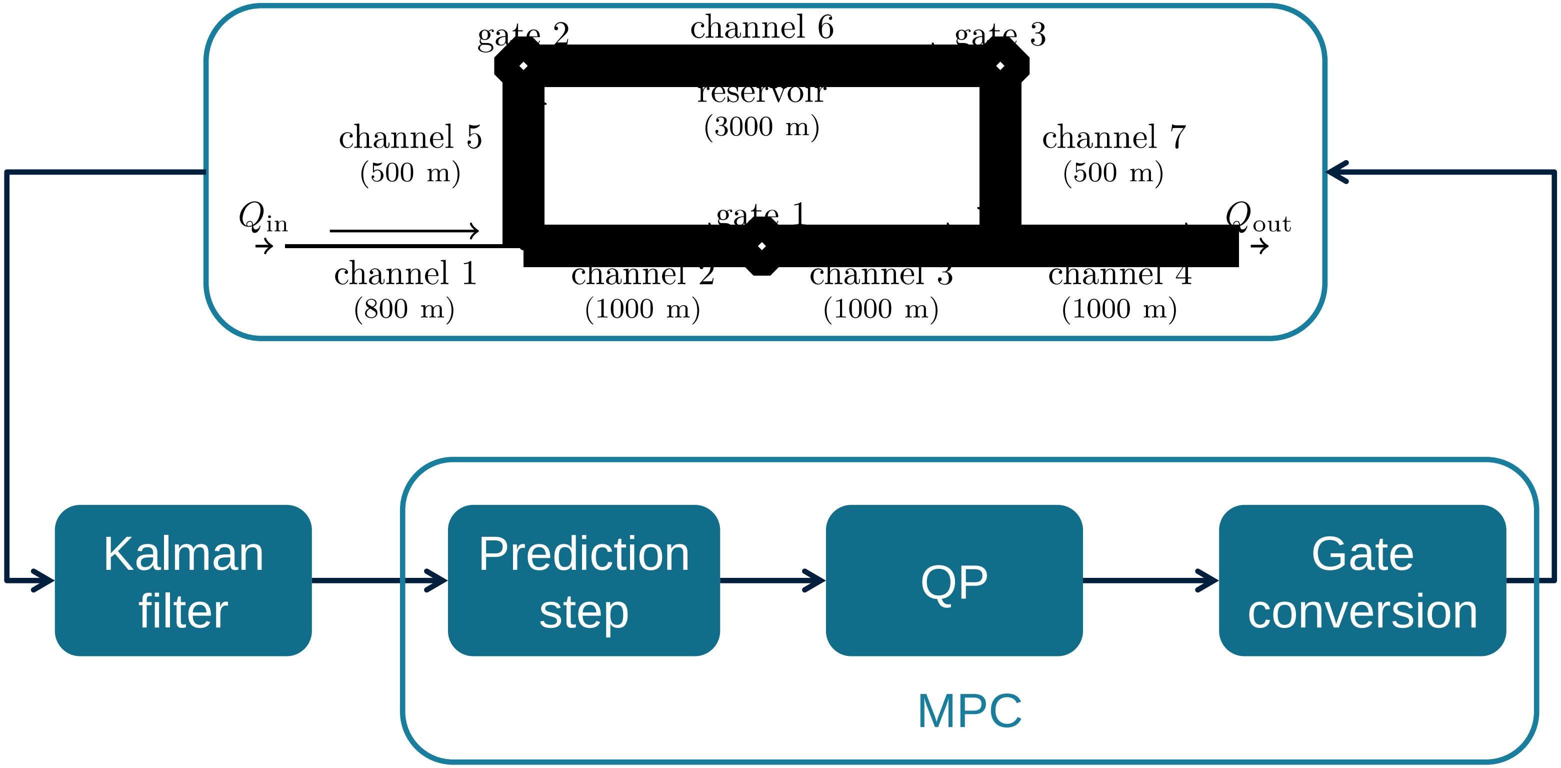
$$\min_{B,S} \sum_{i=1}^{m} \left(A_i - \left(h_i B + h_i^2 S\right)\right)^2$$

s.t. $B \geq 0,\ S \geq 0.$



KU LEUVEN

Model Predictive Control & artificial test example

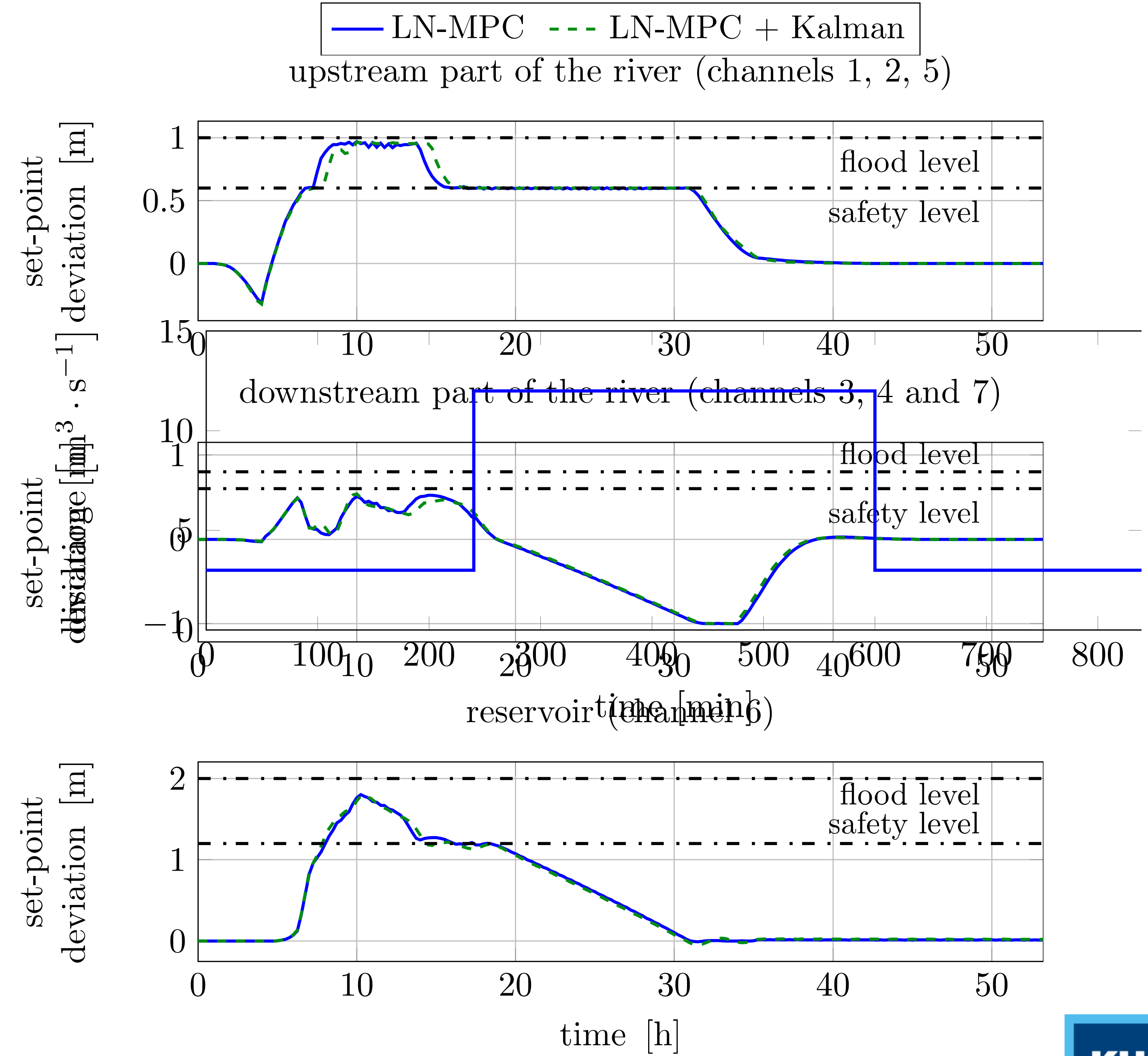


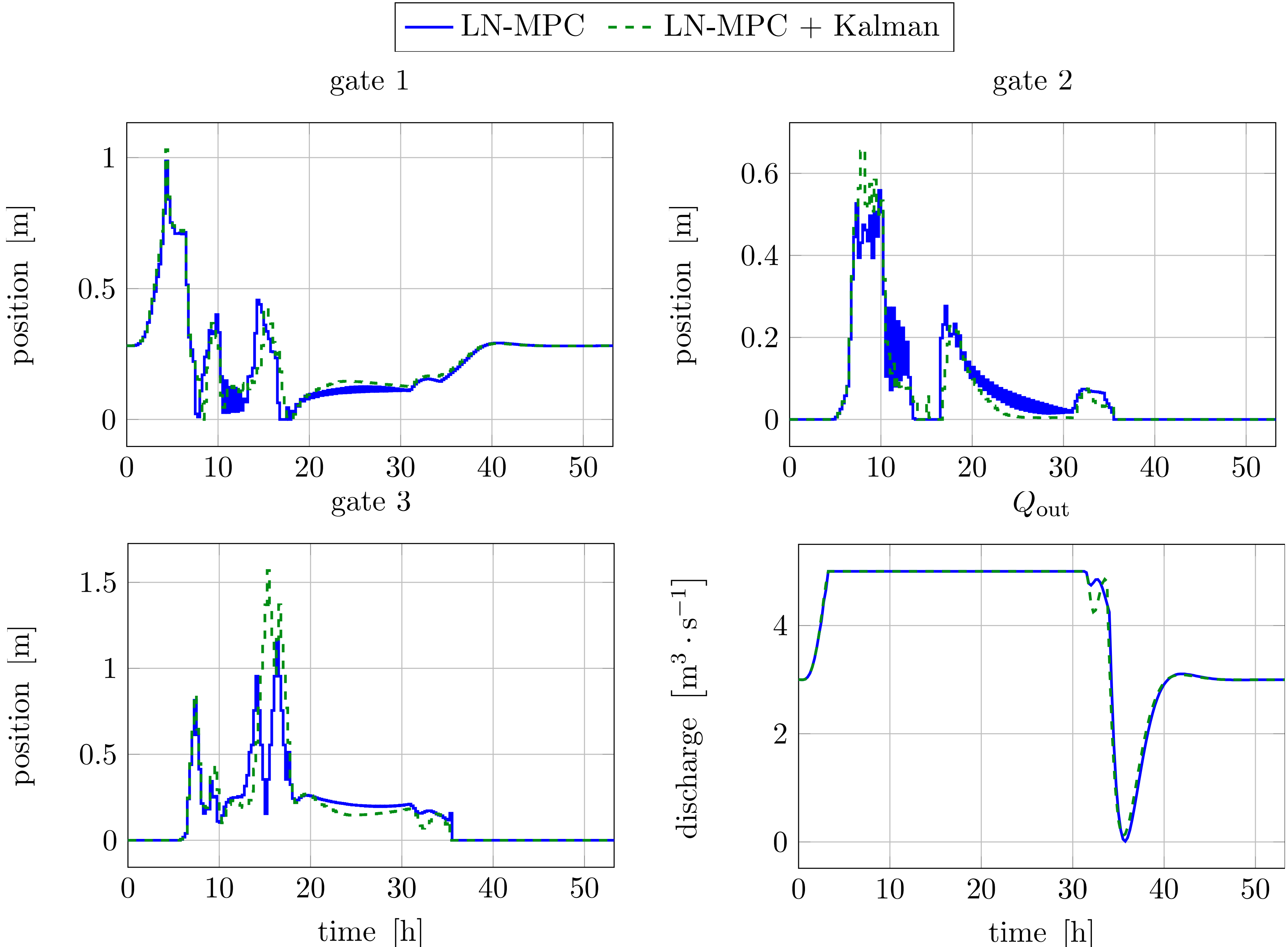
Simulation results

	channel 1	channel 2	channel 3	channel 4	channel 5	reservoir	channel 7
$\mathbf{W} \in \mathbb{R}^{333\times333}$							
water levels	$10 \cdot \mathbf{1}_{17}$	$0.001 \cdot \mathbf{1}_{21}$	$0.001 \cdot \mathbf{1}_{21}$	$1 \cdot \mathbf{1}_{21}$ $800 \cdot \mathbf{1}_{21}^{(*)}$	$0.001 \cdot \mathbf{1}_{11}$	$1000 \cdot \mathbf{1}_{61}$	$0.001 \cdot \mathbf{1}_{11}$
discharges	$0.001 \cdot \mathbf{1}_{18}$	$0.001 \cdot \mathbf{1}_{22}$	$0.001 \cdot \mathbf{1}_{22}$	$0.001 \cdot \mathbf{1}_{22}$	$0.001 \cdot \mathbf{1}_{12}$	$0.001 \cdot \mathbf{1}_{62}$	$0.001 \cdot \mathbf{1}_{12}$
$\mathbf{S} \in \mathbb{R}^{7\times7}$							
safety levels	10^5	10^5	10^5	10^5	10^5	10^6	10^5
$\mathbf{s} \in \mathbb{R}^{7\times1}$							
safety levels	10^5	10^5	10^5	10^5	10^5	10^6	10^5
$\mathbf{V} \in \mathbb{R}^{7\times7}$							
flood levels	10^8	10^8	10^8	10^8	10^8	10^8	10^8
$\mathbf{v} \in \mathbb{R}^{7\times1}$							
flood levels	10^8	10^8	10^8	10^8	10^8	10^8	10^8

	$Q_{\text{gate}}^{(1)}$	$Q_{\text{gate}}^{(2)}$	$Q_{\text{gate}}^{(3)}$	Q_{out}
$\mathbf{R} \in \mathbb{R}^{4\times4}$				
control actions	175	175	175	10

KU LEUVEN


LN-MPC
LN-MPC + Kalman
upstream part of the river (channels 1, 2, 5)
set-point deviation [m]
flood level
safety level
downstream part of the river (channels 3, 4 and 7)
set-point deviation [m]
flood level
safety level
reservoir (channel 6)
set-point deviation [m]
flood level
safety level
time [h]


LN-MPC
LN-MPC + Kalman
gate 1
gate 2
gate 3
Q_{out}
position [m]
discharge [m^3 · s^-1]
time [h]
0
0.2
0.4
0.5
0.6
1
1.5
2
4
10
20
30
40
50

Simulation results

	maximal flooding (m)		average set-point deviation (m)	
	LN-MPC	LN-MPC + Kalman	LN-MPC	LN-MPC
channel 1	0.0081	0.0090	$1.51 \cdot 10^{-5}$	$1.10 \cdot 10^{-5}$
channel 2	0.0018	0.0030	$1.54 \cdot 10^{-5}$	$1.10 \cdot 10^{-5}$
channel 3	-0.2735	-0.2591	$2.31 \cdot 10^{-5}$	$7.09 \cdot 10^{-6}$
channel 4	-0.2738	-0.2583	$2.27 \cdot 10^{-5}$	$6.88 \cdot 10^{-6}$
channel 5	-0.0080	-0.0126	$1.56 \cdot 10^{-5}$	$1.14 \cdot 10^{-5}$
channel 6 (reservoir)	-0.1992	-0.2057	0.01	0.02
channel 7	-0.2742	-0.2582	$2.24 \cdot 10^{-5}$	$6.48 \cdot 10^{-6}$

KU LEUVEN